YOU MUST MEMORIZE THESE!

Problems relevant to the following formulas appear on PSAT/SAT and Pre-ACT/ACT Mathematics Tests. Memorizing these formulas and knowing how to apply them will ensure success on the tests.

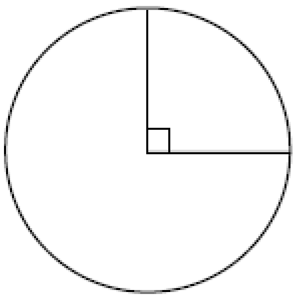
** indicates not frequently tested but necessary for score of 33-36 or 1500+

I. CIRCLES

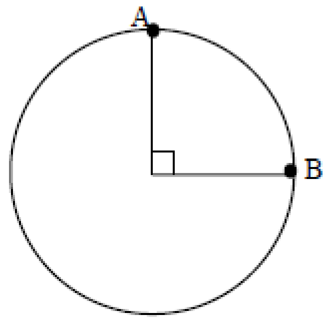
A. CIRCUMFERENCE: $C = 2\pi r$ or $C = \pi d$ (when $\pi = 3.14$ or $\frac{22}{7}$)

B. AREA: $A=\pi r^2$

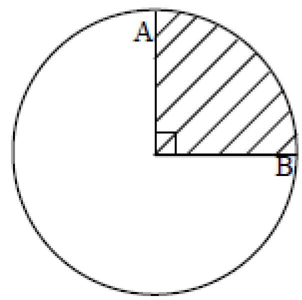
C. EQUIVALENT RATIOS: **Central angle, Length of arc, Area of sector**



$90° = \frac{1}{4}$ of 360°



Length from A to B $= \frac{1}{4}$ of circle's circumference



Area of Sector $= \frac{1}{4}$ of circle's area

D. $360° = 2\pi$ radians

E. Converting degrees to radians

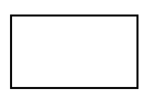
multiply by $\frac{\pi}{180°}$

Converting radians to degrees

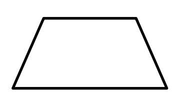
multiply by $\frac{180°}{\pi}$

II. TWO-DIMENSIONAL FIGURES

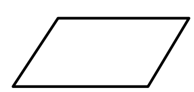
A. RECTANGLE: Area = length x width
$A = lw$



B. TRAPEZOID: Area = $\frac{1}{2}$ height x sum of the bases
$A = \frac{1}{2}h(b_1+b_2)$



C. PARALLELOGRAM: Area = length of base x height
$A = bh$

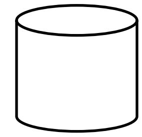


D. SUM OF ANGLES IN FOUR-SIDED FIGURE = 360° or $(n-2)180°$

III. THREE-DIMENSIONAL FIGURES

A. CYLINDERS

1. VOLUME: $V = \pi r^2 h$ where r is the radius and h is the height



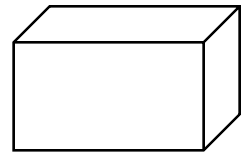
B. RECTANGULAR SOLIDS

1. VOLUME: Volume = length x width x height

   $V = lwh$

2. SURFACE AREA = AREA OF THE SURFACES

   SA = area of the front + area of back + area of top + area of bottom + area of left side + area of right side

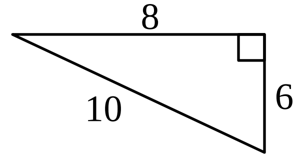


IV. TRIANGLES

A. AREA OF "GARDEN VARIETY" TRIANGLE

$A = \frac{1}{2} bh$

B. AREA OF RIGHT TRIANGLE



Quite Often Area = $\frac{1}{2}$(LEG)(LEG)

$A = \frac{1}{2}(6)(8) = 24$

C. AREA OF EQUILATERAL TRIANGLE

$A = \frac{s^2\sqrt{3}}{4}$

V. TRIANGLES – THOSE THAT ARE RIGHT

A. $a^2 + b^2 = c^2$ Pythagorean Theorem

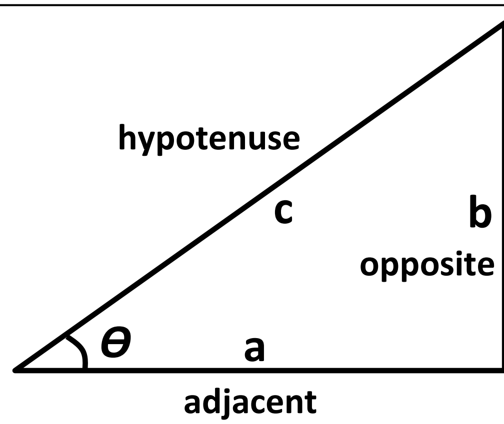
1. Pythagorean Triplets
   - 3-4-5
   - 6-8-10
   - 5-12-13
   - 7-24-25
   - 8-15-17

B. SIN, COS, TAN or "CHIEF SOHCAHTOA"

$\text{SIN } \theta = \frac{\text{Leg Opposite}}{\text{Hypotenuse}}$

$\text{COS } \theta = \frac{\text{Leg Adjacent}}{\text{Hypotenuse}}$

$\text{TAN } \theta = \frac{\text{Leg Opposite}}{\text{Leg Adjacent}}$



SIN ∠ A = COS ∠ B
COS ∠ A = SIN ∠ B
$\text{COT } \angle A = \frac{1}{\text{TAN } \angle A}$

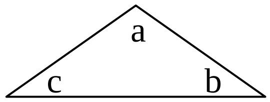
$\text{SIN}^2 + \text{COS}^2 = 1$

$\text{TAN} = \frac{\text{SIN}}{\text{COS}}$

SIN 90° = 1
COS 0° = 1
TAN 45° = 1

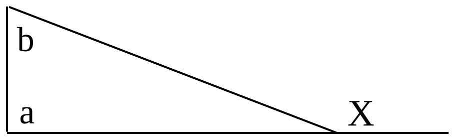
VI. TRIANGLES – ADDITIONAL "GOTTA KNOWS"

A.

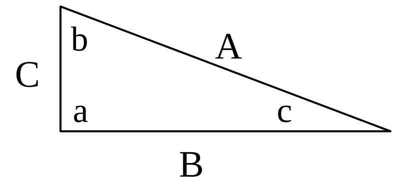


Sum of angles = a + b + c = 180°

B.



Measure of exterior angle = sum of measures of 2 remote interior angles
X = a + b

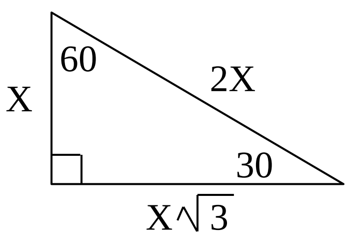


C. Longest side is opposite largest angle
If c < b < a, then C < B < A

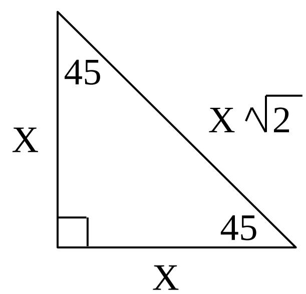
D. The sum of the lengths of <u>ANY</u> two sides of a triangle must be greater than the length of the third side.
A + B > C or A + C > B or B + C > A

VII. TRIANGLES THAT ARE SO SPECIAL

30° - 60° - 90° TRIANGLE

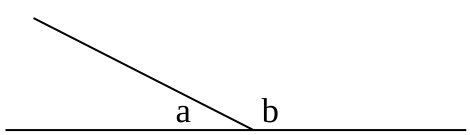


45° - 45° - 90° TRIANGLE

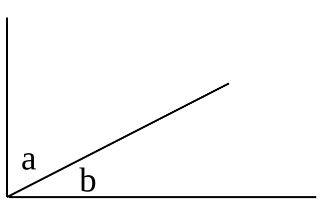


***IMPORTANT COROLLARY**
Length of side opposite the 30° is $\frac{1}{2}$ the length of the hypotenuse

VIII. ANGLE SUMS



SUM OF SUPPLEMENTARY ∠'S = 180°
a + b = 180°



SUM OF COMPLIMENTARY ∠'S = 90°
a + b = 90°

IX. SLOPE UPSIDE DOWN, BACKWARD AND FORWARD

A. STANDARD FORM EQUATION

$$Ax+By+C = 0$$

$$\text{slope} = -\frac{A}{B}$$

B. SLOPE INTERCEPT FORM EQUATION

$$y = mx+b$$

m = slope of the line

b = y-intercept; i.e. (0, b)

C. POINT-SLOPE FORM EQUATION

$$y - y_1 = m(x - x_1)$$

D. SLOPE $= \dfrac{y_2 - y_1 \text{ (rise)}}{x_2 - x_1 \text{ (run)}}$

X. AVERAGES

IF AN AVERAGE IS GIVEN IN A PROBLEM:

THE SUM OF THE ITEMS = THE NUMBER OF ITEMS TIMES THE AVERAGE

$$S = NA$$

XI. OTHER MUSTS

A. QUADRATIC FORMULA $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

B. MIDPOINT FORMULA $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

C. DISTANCE BETWEEN TWO POINTS FORMULA

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

D. DISTANCE TRAVELED FORMULA

DISTANCE TRAVELED = RATE TIMES TRAVEL TIME

$$D = RT$$

XII. BASIC PATTERNS

A. COMPLEX NUMBERS

$$i = \sqrt{-1}$$

$$i^2 = -1$$

B. FOILING AND FACTORING

Multiply: $(x+a)(x+b) = x^2 + bx + ax + ab$

(first terms; outside terms; inside terms; last terms)

Important quadratics to know how to factor:

$x^2 + 2xy + y^2 = (x + y)(x + y) = (x + y)^2$

$x^2 - 2xy + y^2 = (x - y)(x - y) = (x - y)^2$

$x^2 - y^2 = (x + y)(x - y)$

Completing the square:

$x^2 + 6x + 7$

$\left(x^2 + 6x + (\frac{6}{2})^2\right) + 7 - (\frac{6}{2})^2$

$(x^2 + 6x + 9) + 7 - 9$

$(x+3)^2 - 2$

C. EXPONENTS

1. When the bases are the same, to multiply – add the exponents

$(x^2)(x^6) = x^8$

2. When the bases are the same, to divide – subtract the exponents

$\frac{x^8}{x^6} = x^2$

3. To raise to a power – multiply exponents

$(x^2)^6 = x^{12}$

4. Negative exponents: perform the normal operation (raise the quantity to the power) and then INVERT

$2^3 = 8$, THEREFORE $2^{-3} = \frac{1}{8}$, $(x+2)^{-2} = \frac{1}{(x+2)^2}$

5. Fractional exponents: convert into a radical expression

$x^{\frac{2}{3}} = \sqrt[3]{x^2}$

6. Expanding exponents

$9^x = 3^{2x}$

D. LOGARITHMS

1. $\log_a(x) = N$ means that $a^N = x$

$\log_2(32) = 5$

$2^5 = 32$

2. $\log_b(x) - \log_b(y) = \log_b(x \div y)$

$\log_2(24) - \log_2(3) = \log_2(24 \div 3) = \log_2(8)$

3. $\log_b(x) + \log_b(y) = \log_b(x \cdot y)$

$\log_2(8) + \log_2(4) = \log_2(8 \cdot 4) = \log_2(32)$

4. $2\log_b(x) = \log_b(x^2)$

$3\log_2(4) = \log_2(4^3)$

XIII. METHODS FOR SOLVING

A. SYSTEMS OF EQUATIONS

Can be solved by either substitution or linear combination

SOLVE:

$$\begin{aligned} 2x + y &= 15 \\ 4x + 3y &= 37 \end{aligned}$$

By Substitution:

Step 1: $2x + y = 15 \longrightarrow y = 15 - 2x$
Step 2: (substitution) $4x + 3(15 - 2x) = 37$
Step 3: (distribute) $4x - 6x + 45 = 37$
Step 4: (subtract 45 from both sides) $-2x = -8$
Step 5: (multiply both sides by –1) $2x = 8$
Step 6: (divide both sides by 2) $x = 4$
Step 7: substitute back into the original equation
$2(4) + y = 15$
Step 8: $y = 7$

By Linear Combination:

Step 1: multiply $2x + y = 15$ by $-2 \longrightarrow -4x - 2y = -30$
Step 2: add

$$\begin{aligned} -4x - 2y &= -30 \\ \underline{4x + 3y} &\underline{= 37} \\ y &= 7 \end{aligned}$$

Step 3: substitute $y = 7$ back into the original equation
Step 4: using properties of equality, solve for x:
$2x + 7 = 15 \longrightarrow 2x = 8 \longrightarrow x = 4$

B. EVALUATING FUNTIONS

If $f(x) = 2x+10$, what is $f(-4)$?
use substitution:
$f(-4) = 2(-4)+10 = 2$

C. MATRICES

1. TRANSLATING LINEAR EQUATIONS

$$5x + 3y = 20$$
$$x - 2y = 9$$

$$\left[\begin{array}{cc|c} 5 & 3 & 20 \\ 1 & -2 & 9 \end{array}\right]$$

2. ADDING AND SUBTRACTING – Add or subtract numbers in the matching position

$$\begin{bmatrix} 3 & 8 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & -8 \end{bmatrix} = \begin{bmatrix} 5 & 13 \\ 4 & -6 \end{bmatrix}$$

3. DISTRIBUTING

$$3\begin{bmatrix} 2 & 4 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 15 & 21 \end{bmatrix}$$

4. MULTIPLYING – The number of columns of the 1st matrix must equal the number of rows of the 2nd matrix. The result will have the same number of rows as the 1st matrix and the same number of columns as the 2nd matrix.

$$\begin{bmatrix} 2 & -5 & 1 \\ -3 & 4 & 6 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 3 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 16 & -11 \\ 32 & 36 \end{bmatrix}$$

$(2)(2) + (-5)(-1) + (1)(7) = 16$ $(2)(0) + (-5)(3) + (1)(4) = -11$

$(-3)(2) + (4)(-1) + (6)(7) = 32$ $(-3)(0) + (4)(3) + (6)(4) = 36$

XIV. GRAPHS

A. INEQUALITIES

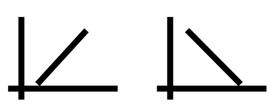
> or ≥ shade above

< or ≤ shade below

B. LINEAR

$f(x) = a + bx$

graph is a straight line

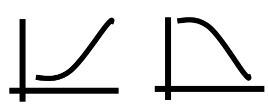


set slope – increases or decreases by the same amount

C. EXPONENTIAL

$f(x) = ab^x + c$

graph is a curved line



changing slope – increases or decreases by the same percentage

**slope: positive if a>0, negative if a<0

**y-intercept: change x to 0 and solve for f(x)

**horizontal asymptote: y = c

D. TRANSFORMATIONS

- f(x - a) = f(x) shifted a units to the right
  - (x – 2) = 0; x = 2
- f(x + a) = f(x) shifted a units to the left
  - (x + 2) = 0; x = -2
- f(x) + a = f(x) shifted a units up
- f(x) - a = f(x) shifted a units down
- -f(x) = f(x) reflected about the x-axis
- f(-x) = f(x) reflected about the y-axis

Remember:
add to y, go high;
add to x, go left

E. CIRCLES

General Equation is $x^2 + y^2 = r^2$ or $(x – h)^2 + (y – k)^2 = r^2$
where r is the radius and (h, k) is the center

F. PARABOLAS

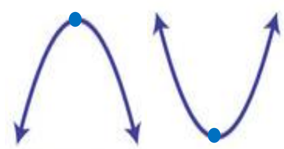
Standard form $y = ax^2 + bx + c$

x-coordinate of the vertex can be found by the formula $\frac{-b}{2a}$

Vertex form $y = a(x – h)^2 + k$**, vertex is the point (h, k)**

Maximum – vertex of negative – opens downward

Minimum – vertex of positive – opens upward



G. ELLIPSES

wider-than-tall ellipse with center at (*h*, *k*)

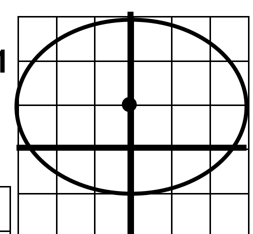
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

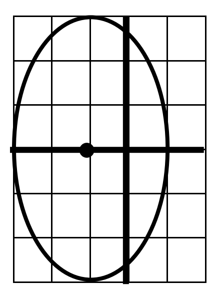
taller-than-wide ellipse with center at (*h*, *k*)

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

***a* & *b* are units from center**
a^2 always goes with the variable whose axis parallels the wider direction of the ellipse; a>b

$$\frac{(x-0)^2}{3^2}+\frac{(y-1)^2}{2^2}=1$$

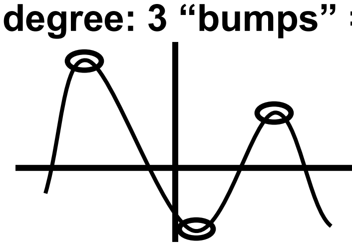




$$\frac{(x+1)^2}{2^2}+\frac{(y-0)^2}{3^2}=1$$

H. POLYNOMIALS

1. **The # of "bumps" is one less than the minimum degree: 3 "bumps" = x^4**



**2. $\frac{ax^m+\ldots}{bx^n+\ldots}$

horizontal asymptote:

n<m → x-axis n=m → $y=\frac{a}{b}$ n>m → none = slant asymptote

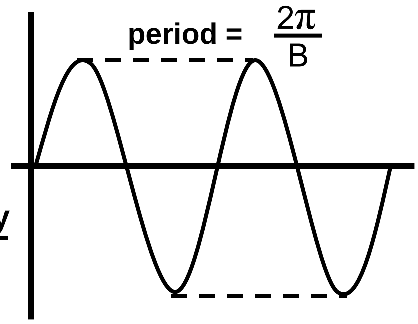
vertical asymptote: set denominator = 0 and solve for x

**I. PERIODIC FUNCTIONS

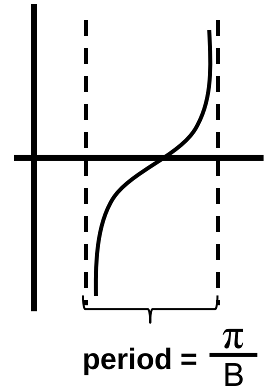
$$y = A \begin{matrix}\sin \\ \cos \\ \tan\end{matrix} (Bx + C) + D$$

amplitude = |A|
period = horizontal distance of wavelength = $\frac{\text{frequency}}{B}$
horizontal shift = $-\frac{C}{B}$
vertical shift = D

SIN/COS



TAN



XV. SERIES PROBLEMS

Arithmetic Sequence – the difference of each term is a number. Use the formula $t_n = t_1 + d(n - 1)$ where t_1 is the first term and d is the common difference.

For example, find the 100th term in the sequence 3, 7, 11, 15 …

$t_{100} = 3 + 4(99)$
$t_{100} = 399$

Geometric Sequence – where the terms are multiplied. Use the formula $t_n = (t_1)(r^{n-1})$ where t_1 is the first term and r is the common ratio.

For example, find the 10th term in the sequence 5, 10, 20, 40 …

$t_{10} = (5)(2^{10-1})$
$t_{10} = (5)(512)$
$t_{10} = 2560$

Summation (Arithmetic) – S_n of the first n terms

$Sn = \frac{n}{2}(a_1 + a_n)$ **where n is a positive integer**

Summation (Geometric)

$S_n = \frac{a_1(1 - r^n)}{1 - r}$ **where r is not equal to 1**