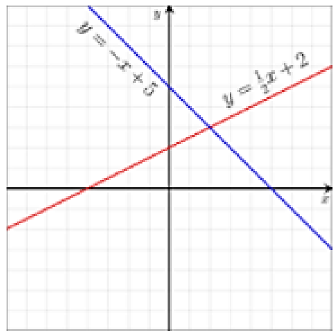
Algebra

Linear Equations & Functions

There will be at least 5-6 questions on linear equations and functions on every ACT test, so this is a very important section to know.

Slope



Slope is the measure of how a line changes. It's expressed as: the change along the y-axis/the change along the x-axis, or $\frac{\text{rise}}{\text{run}}$.

- Given two points, $A(x_1, y_1)$, $B(x_2, y_2)$, find the slope of the line that connects them:

$$\frac{(y_2 - y_1)}{(x_2 - x_1)}$$

Slope-intercept form

- A linear equation is written as $y = mx + b$
  - *m* is the slope and *b* is the y-intercept (the point of the line that crosses the y-axis)
  - A line that passes through the origin (y-axis at 0), is written as $y = mx$
  - If you get an equation that is NOT written this way (i.e. $mx - y = b$), re-write it into $y = mx + b$

Midpoint formula

- Given two points, $A(x_1, y_1)$, $B(x_2, y_2)$, find the midpoint of the line that connects them:

$$\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$$

Good to know

Distance formula

- Find the distance between the two points

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- **You don't actually need this formula,** as you can simply graph your points and then create a right triangle from them. The distance will be the hypotenuse, which you can find via the pythagorean theorem

Logarithms

There will usually only be 1 question on the test involving logarithms. If you're worried about having to memorize too many formulas, don't worry about logs unless you're trying for a perfect score.

$log_b x$ asks "to what power does *b* have to be raised to result in *x*?"

- Most of the time on the ACT, you'll just need to know how to re-write logs

$$log_b x = y => b^y = x$$

$$log_b xy = log_b x + log_b y$$

$$log_b \frac{x}{y} = log_b x - log_b y$$

Statistics and Probability

Averages

The average is the same thing as the mean

- Find the average/mean of a set of terms (numbers)

$$\text{Mean} = \frac{\text{sum of the terms}}{\text{the number(amount) of different terms}}$$

- Find the average speed

$$\text{Speed} = \frac{\text{total distance}}{\text{total time}}$$

Probabilities

Probability is a representation of the odds of something happening. A probability of 1 is guaranteed to happen. A probability of 0 will never happen.

$$\text{Probability of an outcome happening} = \frac{\text{number of desired outcomes}}{\text{total number of possible outcomes}}$$

- Probability of two, mutually exclusive, outcomes *both* happening is

$$\text{Probability of event A} * \text{probability of event B}$$

- e.g., Event A has a probability of $\frac{1}{4}$ and event B has a probability of $\frac{1}{8}$. The probability of both events happening is: $\frac{1}{4} * \frac{1}{8} = \frac{1}{32}$. There is a 1 in 32 chance of *both* events A and event B happening.

Combinations

The possible amount of different combinations of a number of different elements

- A "combination" means the order of the elements doesn't matter (i.e. a fish entree and a diet soda is the same thing as a diet soda and a fish entree)
  - Possible combinations = number of element A * number of element B * number of element C....
  - e.g. In a cafeteria, there are 3 different dessert options, 2 different entree options, and 4 drink options. How many different lunch combinations are possible, using one drink, one, dessert, and one entree?
    - The total combinations possible = 3 * 2 * 4 = 24

Percentages

- Find *x* percent of a given number *n*

$$n\left(\frac{x}{100}\right)$$

- Find out what percent a number *n* is of another number *m*

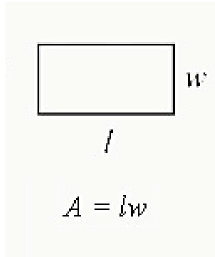
$$\frac{(100n)}{m}$$

- Find out what number *n* is *x* percent of

$$\frac{(100n)}{x}$$

Geometry

Rectangles



Area

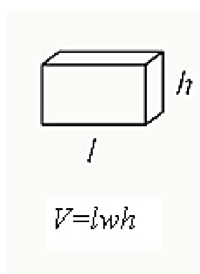
$$\text{Area} = l\,w$$

- *l* is the length of the rectangle
- *w* is the width of the rectangle

Perimeter

$$\text{Perimeter} = 2l + 2w$$

Rectangular solid



Volume

$$\text{Volume} = l\,wh$$

- *h* is the height of the figure

Parallelogram

An easy way to get the area of a parallelogram is to drop down two right angles for heights and transform it into a rectangle.

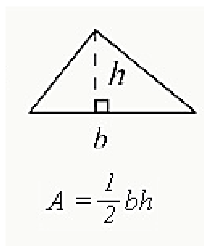
- Then solve for *h* using the pythagorean theorem

Area

$$\text{Area} = l\,h$$

- (This is the same as a rectangle's *lw*. In this case the height is the equivalent of the width)

Triangles



Area

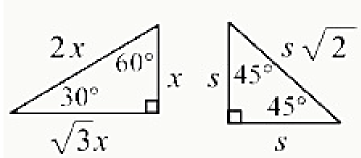
$$\text{Area} = \frac{1}{2}bh$$

- *b* is the length of the base of triangle (the edge of one side)
- *h* is the height of the triangle
  - The height is the same as a side of the 90 degree angle in a right triangle. For non-right triangles, the height will drop down through the interior of the triangle, as shown in the diagram.

Pythagorean theorem

$$a^2 + b^2 = c^2$$

- In a right triangle, the two smaller sides (a and b) are each squared. Their sum is the equal to the square of the hypotenuse (c, longest side of the triangle)



Special Right Triangles

Properties of special right triangle: isosceles triangle

- An isosceles triangle has two sides that are equal in length and two equal angles opposite those sides
- An isosceles right triangle always has a 90 degree angle and two 45 degree angles.
- The side lengths are determined by the formula: $x, x, x\sqrt{2}$, with the hypotenuse (side opposite 90 degrees) having a length of one of the smaller sides * $\sqrt{2}$.
  - E.g., An isosceles right triangle may have side lengths of 12, 12, and $12\sqrt{2}$

Properties of special right triangle: 30, 60, 90 degree triangle

- A 30, 60, 90 triangle describes the degree measures of its three angles
- The side lengths are determined by the formula: x, $x\sqrt{3}$, and $2x$.
  - The side opposite 30 degrees is the smallest, with a measurement of x
  - The side opposite 60 degrees is the middle length, with a measurement of $x\sqrt{3}$
  - The side opposite 90 degree is the hypotenuse, with a length of $2x$
  - For example, a 30-60-90 triangle may have side lengths of 5, $5\sqrt{3}$, and 10.

Trapezoids

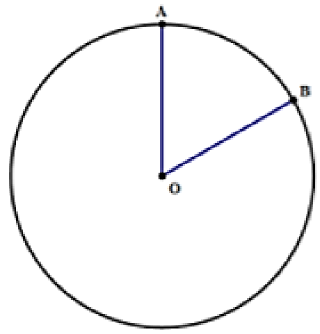
Area

- Take the average of the length of the parallel sides and multiply that by the height

$$\text{Area} = \left[\frac{(\text{parallel side a} + \text{parallel side b})}{2}\right]h$$

- Often, you are given enough information to drop down two 90 angles to make a rectangle and two right triangles. You'll need this for the height anyway, so you can simply find the areas of each triangle and add it to the area of the rectangle, if you would rather not memorize the trapezoid formula
- Trapezoids and the need for a trapezoid formula **will be at most one question on the test**. Keep this as a minimum priority if you're feeling overwhelmed.

Circles



Area

$$\text{Area} = \pi r^2$$

- π is a constant that can, for the purposes of the ACT, be written as 3.14 (or 3.14159)
    - Especially useful to know if you don't have a calculator that has a π feature or if you're not using a calculator on the test.
- r is the radius of the circle (any line drawn from the center point straight to the edge of the circle)

Area of an arc

- Given a radius and a degree measure of an arc from the center, find the circumference of the arc
- Use the formula for the area multiplied by the angle of the arc divided by the total angle measure of the circle

$$\textit{Area} \text{ of an arc} = (\pi r^2)\left(\text{degree measure of center of } \frac{\text{arc}}{360}\right)$$

Circumference

$$\text{Circumference} = 2\pi r$$

or

$$\text{Circumference} = \pi d$$

- d is the diameter of the circle. It is a line that bisects the circle through the midpoint and touches two ends of the circle on opposite sides. It is twice the radius.

Circumference of an arc

- Given a radius and a degree measure of an arc from the center, find the circumference of the arc
- Use the formula for the circumference multiplied by the angle of the arc divided by the total angle measure of the circle (360)

$$\text{Circumference of an arc} = (2\pi r)\left(\text{degree measure center of }\frac{\text{arc}}{360}\right)$$

  - Example: A 60 degree arc has $\frac{1}{6}$ of the total circle's circumference because $\frac{60}{360} = \frac{1}{6}$

An alternative to memorizing the "formulas" for arcs is to just stop and think about arc circumferences and arc areas logically.

  - If you know the formulas for the area/circumference of a circle and you know how many degrees are in a circle, put the two together.
    - If the arc spans 90 degrees of the circle, it must be $\frac{1}{4}$th the total area/circumference of the circle, because $\frac{360}{90} = 4$.
    - If the arc is at a 45 degree angle, then it is $\frac{1}{8}$th the circle, because $\frac{360}{45} = 8$.
  - The concept is exactly the same as the formula, but it may help you to think of it this way instead of as a "formula" to memorize.

Equation of a circle

- Useful to get a quick point on the ACT, but don't worry about memorizing it if you feel overwhelmed; **it will only ever be worth one point.**
- Given a radius and a center point of a circle (h, k)

$$(x - h)^2 + (y - k)^2 = r^2$$

Cylinder

$$\text{Volume} = \pi r^2 h$$